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Pareto Improving Third-degree Price Discrimination with Network Effects☆

Ryo Hashizume[a,*] Takeshi Ikeda[a] and Tatsuhiko Nariu[b]

Abstract

This paper analyzes the welfare effects of third-degree price discrimination by a monopoly selling a network good in two separate markets. Through positive network effects, consumers' utility rises as the number of users (i.e., total output) increases. This feature of network effects brings about an unfamiliar welfare consequence of price discrimination: Pareto improving third-degree price discrimination no longer requires that prices decrease in both markets. We provide a sufficient condition for Pareto improvement under a general model, consistent with this claim. We then demonstrate a simple example—a linear model, in which two separate markets differ only in their strength of network effects—in which price discrimination can achieve Pareto improvement with below-marginal-cost pricing.

Keywords: Third-degree price discrimination, Network effects, Monopoly, Pareto improvement
JEL Classification: D42, L12

1. Introduction

This paper analyzes the welfare effects of third-degree price discrimination by a monopolist who produces a network good and serves all markets. We consider that consumers benefit from the positive

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effects exhibited by its users' networks encompassing all markets.[1] This is a common feature of markets for computers and software. For instance, while 'Microsoft word' itself is useful for creating documents, the greater the number of users, the greater the benefit to each user from exchanging files through e-mail without printing out on paper. In addition, firms often conduct third-degree price discrimination when they sell these sorts of goods, as exemplified by the lower priced 'academic package' of Microsoft and 'Dell university program' of Dell. Markets for textbooks, mobile phones, and subscription services (e.g., Amazon prime and YouTube premium) also engender network effects, and are frequently sold to students at discounted prices.[2] The significant work of Shapiro and Varian (1988) argues that network effects themselves and the lock-in due to them can be the reason for practicing third-degree price discrimination. In short, third-degree price discrimination and network goods are closely connected. Nevertheless, little of the literature on third-degree price discrimination has dealt with network effects. This is probably because it is not fully recognized that welfare is measured differently depending on whether the network effects are present, and the importance of dealing with the case with network effects is overlooked. In this paper, we shall first clarify the differences between network and non-network goods that affect consumer surplus and social welfare, and then analyze the welfare effects of third-degree price discrimination in the presence of network effects.

What features do network effects introduce to a model with two separate markets? One is demand interdependence. Unless network effects work only within each separate market, each price influences the demand in both markets.[3] That is, in the presence of positive network effects spanning both markets, if the price in one market decreases, causing quantity demanded in that market to increase and the network to expand, the expansion of the network causes users in the other market to value the good more highly and so the demand in the other market must increase. However, demand

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[1] Throughout this paper, we assume that all consumers are not interested in the member of users, only the number of users. See Rohlfs (1974) for earlier study dealing with communities of interest.

[2] People who use same textbooks or subscription services can benefit from sharing thoughts on common topics.

[3] See Adachi (2005) for consumption externalities within separate markets.

interdependence is not the decisive difference between markets with network effects and others in the sense that it is not peculiar to markets with network effects. In fact, Varian (1985, 1989) and Layson (1998) model third-degree price discrimination with demand interdependence but without network effects. The more significant implications of network effects are on how to measure welfare.[4] In a model with network effects, we cannot measure consumer surplus in each market by integrating the demand function that determines the equilibrium. As discussed precisely below, consumer surplus is equivalent to the areas between (inverse) demand curves given a network's size. Based on these features, Adachi (2002, 2004, 2005) and subsequent research (Ikeda and Nariu, 2009; Okada and Adachi, 2013; Czerny and Zhang, 2015) analyze third-degree price discrimination in the presence of consumption externalities, and demonstrate that some welfare results hold in the absence of consumption externalities are no longer established when such externalities are present.[5] Adachi (2002, 2004, 2005) and Czerny and Zhang (2015) show that the well-known necessary condition for improving social welfare — total output is higher under price discrimination than uniform pricing — does not hold in the presence of network effects.[6] Okada and Adachi (2013) demonstrate that price discrimination can worsen social welfare even when it opens up a new market which is closed in the uniform pricing regime. Ikeda and Nariu (2009), in a contribution closely related to this paper, demonstrate that, in the presence of consumer externalities between markets, price discrimination can improve aggregate consumer surplus and improve social welfare. Note that they also touch on the possibility of price discrimination bringing about Pareto improvement, but the condition under which it happens is somewhat restrictive in the sense that the rate of change of the demand for good *i* with respect to the price of market *i* is smaller than that with respect

[4] See Adachi (2002, 2004) and Bertoletti (2004) for the distinction of consumer surplus depending on whether network effects exist.

[5] Note that, in many cases, terms such as "consumption externalities," "network externalities," and "network effects "are used interchangeably. See Chou and Shy (1990), Liebowitz and Margolis (1994) and Farrell and Klemperer (2007) for strict differences between network externalities and network effects.

[6] Robinson (1933) remarks the necessary condition, and then Schmalensee (1981) shows that, under constant marginal cost and independent demands, it holds. Thereafter Varian (1985), Schwartz (1990) and Bertoletti (2004) demonstrate the robustness of the necessary condition under more general models.

to the price of market j: i.e., the cross-price effect dominates the own-price effect. Following from the above research on third-degree price discrimination with network effects, this paper aims to clarify the welfare effects of price discrimination in a situation where a monopolist's good constitutes an industry-wide network, under which consumers have network effects not only between markets but also within each market. Especially, we focus on the sufficient condition for Pareto improvement.

Research on third-degree price discrimination since the seminal work of Pigou (1920) is ample, but very little of it addresses the sufficient conditions for Pareto improvement. This is because moving from uniform pricing to discriminatory pricing typically raises prices in the markets with relatively less elastic demand. Consumers in the markets in which prices increase suffer utility loss with no gain, so Pareto improvement does not occur.[7] However, in the presence of network effects, consumers gain when price discrimination increases the size of the network by expanding total output. Therefore, consumer surplus in a market could improve even if the price increases as long as the network expands. The present paper confirms this conjecture and concludes that prices decreasing in both markets is not a necessary condition for Pareto improving third-degree price discrimination when a monopoly firm's product exhibits a positive network effect.

There are two remarks regarding our analytical method. First, similar to Grilo et al. (2001), Shy (2001), Adachi (2002, 2004, and 2005) and subsequent research discussed above, we assume that a monopoly firm can credibly commit to its actions, and consumers who are influenced by such actions have *perfect foresight* or form a *self-fulfilling* (*rational*) *expectations*. That is, for given prices, consumers anticipate the actual network size and form a correct expectation under which the total demand equals the expectation. Therefore, to derive demand functions, we need to find an expectation that is a fixed point for given prices. With reference to the monotone comparative-static methods used by Kwon (2007) and Amir and Lazatti (2011), we make assumptions needed to apply Topkis's monotonicity theorem and

[7] As exceptions, Leontief (1940), Hausman and Mackie-Mason (1988) and Nahata et al. (1990) for independent markets, Layson (1998) for interdependent market, find the conditions under which price discrimination lowers all markets' prices, so Pareto improving occurs.

Tarski's fixed point theorem for guaranteeing existence of such a fixed point.[8] Then, uniqueness of the point is guaranteed by Gale and Nikaido's (1965) univalence theorem.[9] Second, we adopt the Lagrangian method with a price difference constraint developed by Leontief (1940) and used by Layson (1998) for capturing the continuum of effects by regime change from uniform pricing to discriminatory pricing.[10] It should be noted that we regard outputs instead of prices as monopolist's choice variables for profit maximization, and then the Lagrangian is a function of outputs.[11] The reason for adopting such a roundabout method is the difficulty of obtaining the second derivatives of the direct demand functions, which makes it impossible to derive mathematical conditions ensuring the uniqueness of the Lagrangian solution explicitly. However, instead of pursuing the rigor of logic, we need to make additional assumptions that the price difference constraint will be a convex function to guarantee the uniqueness.

To highlight our contributions, let us clarify the differences from closely related studies such as Layson (1998) and Czerny and Zhang (2015). Layson (1998) makes a fairly general analysis of third-degree price discrimination when markets are interdependent but does not look at the effect on output in each market. We take Layson's research one step further and focus on the change of output in each market, which leads to a sufficient condition for Pareto improvement. Also, we decompose the effect of price discrimination on social welfare into three effects: misallocation effect, total output effect and network effect.[12] By using such decomposition, we present a necessary condition for improving social welfare.

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[8] Kwon (2007) and Amir and Lazatti (2011) find the conditions for the existence of a fulfilled expectations Cournot equilibrium defined by Katz and Shapiro (1985) in each model. See also Milgrom and Roberts (1990), Vives (1990, 1999), Topkis (1998) and Amir (2005) for monotone comparative statics approaches based on lattice-theoretical method.

[9] See Vives (1999) for how to apply the univalence theorem to obtain demand functions from inverse demand functions.

[10] Silberberg (1970), Holmes (1989), Aguirre et al. (2010) Weyl and Fabinger (2013), Czerny and Zhang (2015) and Miklós-Thal and Shaffer (2021) use the same Lagrangian method for analyzing the effects of third-degree price discrimination when the number of markets are two. Schmalensee (1981) also uses the Lagrangian method but impose the different constraint to treat the case with more than two markets.

[11] Cheung and Wang (1994) also proceed their analysis by considering that monopolist chooses outputs although they do not use Lagrangian method.

[12] Following from Ippolito (1980), Aguirre (2008) and Aguirre et al. (2010), we use the term 'misallocation effect'. It is also called 'maldistribution' by Robinson (1933) or 'distribution effect' by Schmalensee (1981).

Our approach is similar to Czerny and Zhang (2015), but there are several clear differences. First, where they consider a negative network effect due to congestion, we deal with a positive one. As a result, in contrast to their demand functions that exhibit substitutability, our demand functions exhibit complementarity. Second, while they focus on the increase or decrease of social welfare, we focus on whether Pareto improvement occurs. Third, they assume that consumers' valuations of a network are represented by constant multiples of a function. In contrast, our general analysis allows that consumers value a network based on different functional forms. Moreover, we refer to the necessary condition and sufficient condition for aggregate consumer surplus to be improved by price discrimination.

The remainder is organized as follows. Section 2 sets out our model and presents associated Lagrangian method. Section 3 proceeds the analysis on a general model, and we clarify what consumer surplus would be in the presence of network effects, and then obtain some welfare results including our main result: a sufficient condition of Pareto improving. Section 4 demonstrates that price discrimination brings about Pareto improvement with below marginal cost pricing under a simple linear model where markets differ only the strength of network effect. Section 5 concludes.

2. The model

Consider a profit-maximizing monopolist selling a network good in two separate markets. We assume that the monopolist produces the good with constant marginal cost c (> 0). Moreover, we only consider the situation where the firm serves both markets. In contrast to Varian (1985), we rule out an inter-connection between markets; we assume that there is no consumer arbitrage or other limitation on the monopolist's third-degree price discrimination.[13] Thus, markets are perfectly separated unless network effects are present. The intensity of the network effect is determined by total output (or total consumption); i.e., an industry-wide network is constituted.

[13] See Varian (1989) and Layson (1998) for the examples of inter-connecting markets by the reasons.

There is a representative consumer in each market i ($i = 1, 2$).[14] Before choosing the level of consumption q_i, each consumer forms an expectation of the future size of the network, S (i.e., expectation of total output). Then, consumers maximize their utility in accordance with given prices, p_i, and based on that expectation. Following Hoernig (2012), who was the first to introduce a representative consumer approach in modelling network effects, we consider that the consumer in market i has the following quasi-linear utility function:[15]

$$U_i(q_i,\ y_i;\ S) = u_i(q_i) + f_i(S)q_i + y_i, \tag{1}$$

where the first term represents the utility from enjoying the intrinsic properties of the good, the second equals the benefit from joining the network per unit $f_i(S)$ times the quantity demanded, and y_i is the quantity of the numeraire good with normalized price 1. We assume that $u_i(q_i)$ is three-times differentiable, strictly increasing, strictly concave, with $u_i(0) = 0$, and $f_i(S)$ is twice differentiable, non-decreasing with $f_i(0) = 0$. With a sufficiently large income, the utility maximization problem is reduced to the maximization of consumer surplus in market i, $CS_i(q_i; p_i, S) = u_i(q_i) + f_i(S)q_i - p_iq_i$. Define $r_i(q_i) \equiv du_i(q_i)/dq_i$, for which first-order conditions are given by

$$p_i = p_i(q_i;\ S) = r_i(q_i) + f_i(S). \tag{2}$$

That is, the price of market i equals the marginal utility of the consumer, the sum of marginal utility of the intrinsic property of the good and the network effects. Alternatively, the inverse demand function is the case of an additive separable network effect, under which much of static models with network effects are analyzed (See Katz and Shapiro (1985) and Economides (1996) among others).[16] Solving the first-order conditions, we obtain each demand function of the own market price and an expectation, $D_i(p_i; S)$.

[14] Unlike the usual practice since Varian (1985), we start with a (direct) utility function rather than indirect utility function tor tractable use likewise Cowan (2007) and Czerny and Zhang (2015).

[15] In recent years, Toshimitsu (2016) and Hashizume and Nariu (2020) among others set up such a representative consumer and conducts research on network effects.

[16] Note that this type of additive separable demand functions cannot capture pure network goods with no intrinsic value such as most telecommunications devices (telephone, fax, and e-mail). See Amir and Lazatti (2011) for more general demand which is applicable to pure network products as well as non-pure network goods. See also Rohlfs (1974) for an earlier research of communications service, which is typical pure network one.

Note that uniqueness of $D_i(p_i; S)$ is guaranteed by the strict concavity of CS_i in q_i.

As described in the previous section, we assume that consumers form a *self-fulfilling expectation*; i.e., consumers form an expectation satisfying $D(p_1, p_2; S) = \sum_{i=1,2} D_i(p_i; S) = S$. To guarantee the existence of such a fixed point $S(p_1, p_2)$, and uniqueness of associated demands defined by $q_i(p_1, p_2) \equiv D_i(p_i; S(p_1, p_2))$, we impose the following assumptions:

(A1) The monopolist has a sufficiently large finite capacity limit $2K$ such that $q_i \leq K$ for $i = 1, 2$.[17]

(A2) The function $\varphi(q_1, q_2) = \begin{bmatrix} r_1(q_1) + f_1(q_1 + q_2) \\ r_2(q_2) + f_2(q_1 + q_2) \end{bmatrix}$ has Jacobian matrix A which is everywhere negative quasi-definite; i.e., $(A + A^T)/2 = \begin{bmatrix} r_1' + f_1' & (f_1' + f_2')/2 \\ (f_1' + f_2')/2 & r_2' + f_2' \end{bmatrix}$ is negative definite where superscript T represents transposition of the matrix.

(A1) enables us to consider that, for given prices, $D(p_1, p_2; S)$ is a function of S from the closed interval (complete in one dimensional Euclidean space) $[0, 2K]$ to itself. Then, because $\partial^2 CS_i / \partial S \partial q_i = f_i' \geq 0$, Topkis's monotonicity theorem implies that $D_i(p_i; S)$ is non-decreasing in S and so $D(p_1, p_2; S)$ is. Therefore, Tarski's fixed-point theorem guarantees the existence of $S(p_1, p_2)$. From the existence of $S(p_1, p_2)$, $q_i(p_1, p_2)$ has at least one value. At the pair of values, $q_1 = q_1(p_1, p_2)$ and $q_2 = q_2(p_1, p_2)$, $p_i = r_i(q_i) + f_i(q_1 + q_2)$ must hold by (2). Thus, if the function $\varphi(q_1, q_2)$ is univalent, the uniqueness of $q_i(p_1, p_2)$ is assured. Considering that we may restrict the domain of $\varphi(q_1, q_2)$ to convex rectangle $[0, K] \times [0, K]$ by (A1), Gale and Nikaido's (1965) theorem assures that $\varphi(q_1, q_2)$ is univalent by (A2). In consequence, we obtain demand functions $q_i(p_1, p_2)$. For the following analysis, let us define $p_i(q_1, q_2)$ as the inverse demand of $q_i(p_1, p_2)$,and list the derivatives of direct and indirect demands:

$$\begin{bmatrix} \partial p_1/\partial q_1 & \partial p_1/\partial q_2 \\ \partial p_2/\partial q_1 & \partial p_2/\partial q_2 \end{bmatrix} = \begin{bmatrix} r_1' + f_1' & f_1' \\ f_2' & r_2' + f_2' \end{bmatrix}, \tag{3}$$

[17] The size of capacity does not influence our result, substantially.

$$\begin{bmatrix} \partial q_1/\partial p_1 & \partial q_1/\partial p_2 \\ \partial q_2/\partial p_1 & \partial q_2/\partial p_2 \end{bmatrix} = \frac{1}{\Omega}\begin{bmatrix} r_2' + f_2' & -f_1' \\ -f_2' & r_1' + f_1' \end{bmatrix}, \tag{4}$$

where $\Omega = (r_1' + f_1')(r_2' + f_2') - f_1' f_2' > 0$ by (A2). Moreover, from (A2) and the monotonicity of f, we have $\partial q_i/\partial p_i < 0$, which implies the law of demand, and $\partial q_j/\partial p_i \leq 0$, $i, j = 1, 2$, $i \neq j$, which implies complementarity between markets. Furthermore, these demand functions satisfy standard properties suggested by Layson (1998), such that own-price effects dominate cross price effects in two respects: $\partial q_i/\partial p_i + \partial q_i/\partial p_j < 0$ and $\partial q_i/\partial p_i + \partial q_j/\partial p_i = \partial Q/\partial p_i < 0$. The former means that if prices rise in both markets, the quantity demanded in each market falls. The latter means that if the price of market i rises, then the total output, $Q = q_1 + q_2$, falls. Here, we impose the following assumption to guarantee the dominance of own-price effect over cross-price effect from another perspective:

(A3) $|\partial q_i/\partial p_i| > |\partial q_i/\partial p_j|$ holds, so $r_i' + f_i' + f_j' < 0$, $i, j = 1, 2$, $i \neq j$. This also implies $|\partial p_i/\partial q_i| > |\partial p_j/\partial q_i|$.

Also, the Hessian matrix of p_i is given as follows:

$$\begin{bmatrix} \partial^2 p_i/\partial q_i^2 & \partial^2 p_i/\partial q_i \partial q_j \\ \partial^2 p_i/\partial q_j \partial q_i & \partial^2 p_i/\partial q_j^2 \end{bmatrix} = \begin{bmatrix} r_i'' + f_i'' & f_i'' \\ f_i'' & f_i'' \end{bmatrix}, \tag{5}$$

Assuming the monopolist knows that consumers form a self-fulfilling expectation, and internalize the network effects.[18] The monopolist's profit is given by

$$\Pi(q_1, q_2) = [p_1(q_1, q_2) - c]q_1 + [p_2(q_1, q_2) - c]q_2. \tag{6}$$

Note that we regard the profit as a function of outputs instead of prices to associate the effect of price discrimination with shapes of r_i and f_i. Now, we impose the following assumptions to ensure the sufficiency of the first-order conditions to solve the maximization problem:

[18] Throughout monopolist's pricing, the magnitude of the network effect is determined in the markets. In this sense, it would not be suitable to use the term 'externalities' in our analysis. See also Katz and Shapiro (1985), Economides (1996), Griva and Vettas (2011) and Hurkens and Lopez (2014) for the case where firms cannot control the consumers' expectations formation, so network externalities arise.

(A4) The Hessian of $\Pi(q_1, q_2)$ is negative definite; i.e., $\Pi_{ii} < 0$ and $\Pi_{11}\Pi_{22} - (\Pi_{12})^2 > 0$ where $\Pi_{ii} = 2\partial p_i/\partial q_i + (\partial^2 p_i/\partial q_i^2)q_i + (\partial^2 p_j/\partial q_i^2)q_j$, $\Pi_{12} = \partial p_1/\partial q_2 + \partial p_2/\partial q_1 + (\partial^2 p_1/\partial q_1\partial q_2)q_1 + (\partial^2 p_2/\partial q_1\partial q_2)q_2$.

Then, from (3) and (5), we can rewrite the second derivatives of Π as follows:

$$\Pi_{ii} = 2(r_i{'} + f_i{'}) + q_i r_i{''} + q_i f_i{''} + q_j f_j{''}, \tag{7}$$

$$\Pi_{12} = f_1{'}(1 + q_1 f_1{''}) + f_2{'}(1 + q_2 f_2{''}). \tag{8}$$

When price discrimination is allowed, the firm chooses the outputs that maximize the profit as shown by the solution to the following first-order conditions:

$$\Pi_i = (p_i - c) + (\partial p_i/\partial q_i)q_i + (\partial p_j/\partial q_i)q_j,$$
$$= (p_i - c) + (r_i{'} + f_i{'})q_i + f_j{'}q_j = 0. \tag{9}$$

Let q_i^d be the optimal output under the regime of discriminatory pricing. Now we rule out the trivial case where price discrimination leads to the same prices for both markets, and assume that $p_1^d > p_2^d$ holds without loss of generality. By (9), this condition is given by

$$r_1{'}(q_1^d)q_1^d < r_2{'}(q_2^d)q_2^d. \tag{10}$$

Now, let us apply the Lagrangian method based on the price difference constraint; $p_1(q_1, q_2) - p_2(q_1, q_2) \leq t$, $t \geq 0$. Given the constraint, the Lagrangian function for the monopolist's profit maximization problem is given by

$$L = \Pi(q_1, q_2) - \lambda[p_1(q_1, q_2) - p_2(q_1, q_2) - t], \tag{11}$$

where λ is a Lagrange multiplier. For $t = 0$, we have the uniform pricing problem. If t is sufficiently large and $t \geq t^* = p_1^d - p_2^d$, then the pair of quantities that maximize L equal the optimal discriminatory outputs, (q_1^d, q_2^d). The effect of regime change from uniform pricing to discriminatory pricing is analyzed through the change of t from 0 to t^*. Note that for any $t \in [0, t^*]$, the constraint is binding, and $\lambda > 0$ if $t \in [0, t^*)$ and $\lambda = 0$ if $t = t^*$. For $t \in [0, t^*]$, the first-order conditions are

$$L_1 \equiv \partial L/\partial q_1 = \Pi_1 - \lambda(\partial p_1/\partial q_1 - \partial p_2/\partial q_1) = 0, \tag{12}$$

$$L_2 \equiv \partial L/\partial q_2 = \Pi_2 - \lambda(\partial p_1/\partial q_2 - \partial p_2/\partial q_2) = 0, \tag{13}$$

$$L_\lambda \equiv \partial L/\partial\lambda = -(p_1 - p_2 - t) = 0. \quad (14)$$

To guarantee the optimality and uniqueness of the solution, we impose the following assumption:

(A5) The function $h(q_1, q_2) \equiv p_1(q_1, q_2) - p_2(q_1, q_2)$ is convex; i.e., the Hessian of h given by

$$H = \begin{bmatrix} r_1'' + f_1'' - f_2'' & f_1'' - f_2'' \\ f_1'' - f_2'' & f_1'' - r_2'' - f_2'' \end{bmatrix}$$ is positive semi-definite.

By (A5), the feasible set $\{(q_1, q_2): h(q_1, q_2) \leq t\}$ is convex. Considering that (A4) is a sufficient condition for the objective function $\Pi(q_1, q_2)$ is strictly concave, the uniqueness of the solution is assured.

Let B be the Hessian of L, then $B = \begin{bmatrix} L_{11} & L_{12} & -h_1 \\ L_{12} & L_{22} & -h_2 \\ -h_1 & -h_2 & 0 \end{bmatrix}$ where $h_i = \partial h/\partial q_i$, $h_{ii} = \partial^2 h/\partial q_i^2$, $h_{12} = \partial^2 h/\partial q_1 \partial q_2$, $L_{ii} = \Pi_{ii} - \lambda h_{ii} < 0$, and $L_{12} = \Pi_{12} - \lambda h_{12}$. Under the assumptions (A4) - (A5), the Lagrangian function L is strictly concave, thus L is also strictly concave under the binding constraint; $h(q_1, q_2) = t$. Hence, $\Phi \equiv \det B = -h_1^2 L_{22} + 2h_1 h_2 L_{12} - h_2^2 L_{11}$ must be non-negative. Moreover, considering that $h_1 = r_1' + f_1' - f_2' < 0$ and $h_2 = f_1' - (r_2' + f_2') > 0$ by (3), and $L_{11}L_{22} - (L_{12})^2 > 0$ by (A4) - (A5), the rows of B are linearly independent. As a result, $\Phi > 0$, which characterizes the signs of each market output effect.

Let us end this section by defining the solution of (12) - (14) at $t = 0$ as (q_1^u, q_2^u). We further analyze this Lagrangian optimization problem in section 3.4.

3. The Effects of price discrimination

3.1. Preliminaries: measurement of consumer surplus

Before analyzing the effects of price discrimination, we need to settle the problem of measuring consumer surplus as mentioned in the introduction. We consider the situation where the monopolist chooses the pair of outputs (q_1^*, q_2^*). Then, the price in each market i is $p_i^* = p_i(q_1^*, q_2^*)$ and the size of network is $S^* = q_1^* + q_2^*$. In this case, consumer surplus in market i is given by

$$CS_i(q_i^*;\ p_i^*,\ S^*) = u_i(q_i^*) + f_i(S^*)q_i^* - p_i^*q_i^*$$

$$= \int_0^{q_i^*}[r_i(q_i) + f_i(S^*) - p_i^*]dq_i$$

$$= \int_0^{q_i^*}[p_i(q_i; S^*) - p_i^*]dq_i, \tag{15}$$

where the last equality follows from (2). That is, we must use the demand function which depends on an expectation, $p_i(q_i; S)$, on behalf of the demand function, $p_i(q_1, q_2)$, under which the firm chooses the outputs, for measuring consumer surplus in market i. This is because, the integration of $p_i(q_1, q_2)$ over q_i, depends on the size of network, S, which varies with q_i. Hence, even if $p_i(q_1, q_2)$ can be replicated from another representative consumer utility with or without network effects, the induced effects regarding welfare are different from each other.[19]

At the end of this subsection, we show that output in both markets is the decisive criterion for determining the utility of each consumer in either market.

Lemma 1: *For any pairs of outputs* (v_1, v_2) *and* (w_1, w_2), *consumer surplus in market i at* (v_1, v_2) *is larger than at* (w_1, w_2) *if and only if* $v_i > w_i$.

Proof: It suffices to show that consumer surplus in market i given by (15) is strictly increasing in q_i^* and does not depend on q_j^*. From (2), the formula in the bracket of the right-hand-side of (13) can be rewritten as follows:

$$p_i(q_i;\ S^*) - p_i^* = [r_i(q_i) + f_i(S^*)] - [r_i(q_i^*) + f_i(S^*)] = r_i(q_i) - r_i(q_i^*).$$

Thus, we have $CS_i(q_i^*) = \int_0^{q_i^*}[r_i(q_i) - r_i(q_i^*)]dq_i = u_i(q_i^*) - r_i(q_i^*)q_i^*$. Therefore, we get

$$\partial CS_i(q_i^*)/\partial q_i^* = r_i(q_i^*) - [r_i(q_i^*) + r_i'(q_i^*)q_i^*] = -r_i'(q_i^*)q_i^* > 0; \quad \partial CS_i(q_i^*)/\partial q_j^* = 0. \quad ■$$

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[19] Adachi (2004) shows that the same demand leads to different welfare result in accordance with the existence of network effects. Also, Hashizume et al. (forthcoming) demonstrate that the same demand functions arise from distinct network structures.

As can be seen from (2), the price increases by the amount of the change in network effects. Therefore, the size of consumer surplus in market i depends on the price minus network effects, $r_i(q_i)$, which is strictly increasing in q_i. Because of this result, outputs increasing in both markets is a sufficient condition for third-degree price discrimination to be Pareto improving.

3.2. The effect on social welfare and its decomposition

For third-degree price discrimination to be Pareto improving, social welfare must increase. Therefore, as a first step in our discovery of a sufficient condition for third-degree price discrimination to be Pareto improving, we will consider the conditions for social welfare to be increased. Let W^d and W^u be social welfare under the regime of discriminatory pricing and under uniform pricing. Then, we have the following equations:

$$W^d - W^u = \sum_i \{[u_i(q_i^d) + f_i(S^d)q_i^d - cq_i^d] - [u_i(q_i^u) + f_i(S^u)q_i^u - cq_i^u]\}$$

$$= \sum_i \{[u_i(q_i^d) + f_i(S^u)q_i^d - cq_i^d] - [u_i(q_i^u) + f_i(S^u)q_i^u - cq_i^u] + [f_i(S^d)q_i^d - f_i(S^u)q_i^d]\}$$

$$= \sum_i \int_{q_i^u}^{q_i^u+\Delta q_i} [p_i(q_i; S^u) - c]dq_i + [f_i(S^d) - f_i(S^u)q_i^d]$$

$$= \underbrace{\int_{q_1^u}^{q_1^u+\Delta q_1} [p_1(q_1; S^u) - p^u]dq_1 + \int_{q_2^u}^{q_2^u+\Delta q_2} [p_2(q_2; S^u) - p^u]dq_2}_{\text{misallocation effect}}$$

$$+ \underbrace{\int_0^{\Delta Q} [p^u - c]dq}_{\text{total output effect}} + \underbrace{[f_1(S^d) - f_1(S^u)]q_1^d}_{\text{network effect in market 1}} + \underbrace{[f_2(S^d) - f_2(S^u)]q_2^d}_{\text{network effect in market 2}}, \quad (16)$$

where $\Delta q_i = q_i^d - q_i^u$ and $\Delta Q = \Delta q_1 + \Delta q_2 = Q^d - Q^u = S^d - S^u$. We will explain this decomposition in two parts. One part is the sum of misallocation effect and total output effect, which are quantified based on the demand functions given the expectation under uniform pricing, S^u. These effects are identical to those of Schmalensee (1981). The misallocation effect is always non-positive because $p_1(q_1^u; S^u) = p_2(q_2^u; S^u) = p^u$ and $p_i(q_i; S^u)$ is decreasing in q_i. The sign of the total output effect equals the sign of the total output change. The other parts are network effects in both markets, which are generated by the change of network size from S^u to S^d. Considering the expectation coincides with total output, the sign of the network effect in each market also equals the sign of the total output change. From these observations, we have a next result.

Proposition 1: *A necessary condition for social welfare to be increased by change of regime from uniform pricing to discriminatory pricing is* $Q^d > Q^u$.

In an industry-wide network, the network effects of each market depend only on the total output, not on the output of each market, so the requirement of Proposition 1 is stated in terms of the total output.[20]

3.3. The effect on aggregate consumer surplus

Next, we consider the conditions for aggregate consumer surplus to be increased by price discrimination. It is another necessary condition for Pareto improvement and a sufficient condition for social welfare to be increased. By subtracting the variation of monopolist's profit $\sum_i\{[p_i^d q_i^d - cq_i^d] - [p_i^u q_i^u - cq_i^u]\} = \sum_i\{[r_i(q_i^d)q_i^d + f_i(S^d)q_i^d - cq_i^d] - [r_i(q_i^u)q_i^u + f_i(S^u)q_i^u - cq_i^u]\}$ from the first equation of (16), we have

$$CS^d - CS^u = \sum_i\{[u_i(q_i^d) - r_i(q_i^d)q_i^d] - [u_i(q_i^u) - r_i(q_i^u)q_i^u]\}. \quad (17)$$

If Δq_i and Δq_j have the same sign and if it is positive, aggregate consumer surplus increases because consumer surplus in each market increases from Lemma 1. For the case in which Δq_i and Δq_j have different signs, the following bounds are informative. A proof is omitted because we can derive it straightforwardly by using the strict concavity of u_i.[21]

Proposition 2: *The upper and lower bounds of aggregate consumer surplus are given by*

$$\sum_i\{[r_i(q_i^u) - r_i(q_i^d)]q_i^u\} \leq CS^d - CS^u \leq \sum_i\{[r_i(q_i^u) - r_i(q_i^d)]q_i^d\}. \quad (18)$$

[20] Generally, network effects do not always depend only on the total output, and positive network effect could be generated even if total output is unchanged or decreased. As a result, social welfare can be improved by the discrimination without increasing total output (Adachi 2002, 2005).

[21] These bounds appear in the proof of social welfare bounds in Varian (1985). Against that he uses the convexity of indirect utility function, we use the concavity of direct utility function here.

Therefore, for a change from uniform pricing to discriminatory pricing to increase aggregate consumer surplus, a necessary condition is $\sum_i\{[r_i(q_i^u) - r_i(q_i^d)]q_i^d\} > 0$ *and a sufficient condition is* $\sum_i\{[r_i(q_i^u) - r_i(q_i^d)]q_i^u\} > 0$.

Note that, from (17), the amount of change in aggregate consumer surplus is independent of the network effects function f_i. This is because the amount of change in network effects on both markets are captured by the monopolist. The following equations explains this fact:

$$\Pi^d - \Pi^u = (p_1^d - c)q_1^d + (p_2^d - c)q_2^d - (p^u - c)(q_1^u + q_2^u)$$

$$= (p_1^d - p^u)q_1^d + (p_2^d - p^u)q_2^d + (p^u - c)\Delta Q$$

$$= (r_1^d - r_1^u)q_1^d + (r_2^d - r_2^u)q_2^d$$

$$+ \underbrace{\int_0^{\Delta Q}[p^u - c]dq}_{\text{total output effect}} + \underbrace{[f_1(S^d) - f_1(S^u)]q_1^d}_{\text{network effect in market 1}} + \underbrace{[f_2(S^d) - f_2(S^u)]q_2^d}_{\text{network effect in market 2}}. \quad (19)$$

Also, we can check that the sum of $CS^d - CS^u$ and $\Pi^d - \Pi^u$ equals $W^d - W^u$ from (16), (17), and (19).

3.4. Sufficient conditions for Pareto improvement

Now, let us consider the sufficient conditions Pareto improvement. Alternatively, these are the conditions under which price discrimination increases the output of each market. To accomplish this purpose, we proceed with the Lagrangean method introduced in the previous section.

Totally differentiating (12) - (14) with respect to *t,* we have $B\begin{bmatrix} dq_1/dt \\ dq_2/dt \\ d\lambda/dt \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$. Then, we obtain

$$dq_1/dt = [h_2L_{12} - h_1L_{22}]/\Phi, \quad (20)$$

$$dq_2/dt = [h_1L_{12} - h_2L_{11}]/\Phi, \quad (21)$$

$$d\lambda/dt = -[L_{11}L_{22} - L_{12}^2]/\Phi < 0. \quad (22)$$

Note that, from $\Phi = -h_1^2L_{22} + 2h_1h_2L_{12} - h_2^2L_{11} > 0$, we have $(h_1L_{22}/h_2 - L_{12}) + (h_2L_{11}/h_1 - L_{12}) > 0$. Thus, if $h_1L_{22}/h_2 < h_2L_{11}/h_1$, then L_{12} must be less than h_2L_{11}/h_1, and if $h_1L_{22}/h_2 > h_2L_{11}/h_1$, then L_{12} must be less than h_1L_{22}/h_2, so either $dq_1/dt < 0$ or $dq_2/dt > 0$. Considering that dq_1/dt is increasing in L_{12} and dq_2/dt is decreasing in L_{12}, we obtain the following result.

Lemma 2: (i) *When $h_1L_{22}/h_2 < h_2L_{11}/h_1$ and $dq_2/dt > 0$, then if $L_{12} < h_1L_{22}/h_2$, $dq_1/dt < 0$, and $dq_1/dt > 0$ if $h_1L_{22}/h_2 < L_{12} < h_2L_{11}/h_1$.* (ii) *When $h_1L_{22}/h_2 > h_2L_{11}/h_1$ and $dq_1/dt < 0$, then if $L_{12} < h_2L_{11}/h_1$, $dq_2/dt > 0$, and $dq_2/dt < 0$ if $h_2L_{11}/h_1 < L_{12} < h_1L_{22}/h_2$.*

These results are explained as follows. Let (q_1^t, q_2^t) be profit-maximizing outputs at t. If t rises and the constraint is less tight, λ becomes smaller by (22), so $L_1 = \Pi_1 - \lambda h_1 < 0$ and $L_2 = \Pi_2 - \lambda h_2 > 0$ hold at (q_1^t, q_2^t) because $h_1 < 0$ and $h_2 > 0$ by (A3). Considering that the firm adjusts its quantities to get more profit along with the equation, $h_1 \cdot dq_1/dt + h_2 \cdot dq_2/dt = 1$, which is derived by differentiating the binding constraint $h(q_1, q_2) = t$, $dq_1/dt < 0$ and $dq_2/dt > 0$ holds as long as L_{12} is negative or has a small positive value; i.e., $L_{12} < \min\{h_1L_{22}/h_2, h_2L_{11}/h_1\}$. On the other hand, if $L_{12} > \min\{h_1L_{22}/h_2, h_2L_{11}/h_1\}$, both market outputs increase or decrease. The direction is determined by the size of $-h_1$, h_2, $-L_{11}$, and L_{22}. If h_2 is large and $-L_{22}$ is small, the rise of t induces the change of q_2 to be large. If $-h_1$ is small and L_{11} is large, the rise of t induces the change of q_1 to be small. Therefore, if both h_2 and $-L_{11}$ are large and both $-h_1$ and $-L_{22}$ are small, the effect of the change of q_2 surpasses that of q_1, and both outputs increase. This explanation is represented as a satisfaction of $(-h_1)(-L_{22})/h_2 < h_2(-L_{11})/(-h_1)$. The other direction of output change is explained by the same logic. Following from Lemma 2-(i), we have the sufficient condition for Pareto improvement.

Corollary 1: *A sufficient condition for the regime change from uniform pricing to discriminatory pricing to be Pareto improving is that both $h_1L_{22}/h_2 < h_2L_{11}/h_1$ and $h_1L_{22}/h_2 < L_{12} < h_2L_{11}/h_1$ hold for any $t \in [0, t^*]$.*

In this Corollary, the direction of price changes resulting from a regime change from uniform pricing to discriminatory pricing is not referenced. To find our desired situation where different direction price movements bring about Pareto improvement, we need to know the effects on prices of the change in regime.

In regard to the effects on prices, by totally differentiating $p_i = p_i(q_1, q_2)$ with respect to t, we have $\partial p_i/\partial t = \partial p_i/\partial q_1 \cdot dq_1/dt + \partial p_i/\partial q_2 \cdot dq_2/dt$ and the following equations hold from (20) - (21):

$$dp_1/dt = \{-(\partial p_1/\partial q_1)h_1L_{22} - (\partial p_1/\partial q_2)h_2L_{11} + [(\partial p_1/\partial q_1)h_2 + (\partial p_1/\partial q_2)h_1]L_{12}\}/\Phi, \quad (23)$$

$$dp_2/dt = \{-(\partial p_2/\partial q_1)h_1L_{22} - (\partial p_2/\partial q_2)h_2L_{11} + [(\partial p_2/\partial q_1)h_2 + (\partial p_2/\partial q_2)h_1]L_{12}\}/\Phi. \quad (24)$$

From these equations, we have the conditions under which price discrimination moves the prices in different directions.

Lemma 3: *A sufficient condition for the price of market* 1 *going up is*

$$L_{12} < \{[(-\partial p_1/\partial q_1)h_2]\, h_1L_{22}/h_2 + [(\partial p_1/\partial q_2)(-h_1)]\, h_2L_{11}/h_1\}/[(-\partial p_1/\partial q_1)h_2 + (\partial p_1/\partial q_2)(-h_1)], \quad (25)$$

for any $t \in [0, t^*]$. *A sufficient condition for the price of market* 2 *going down is*

$$L_{12} < \{[(\partial p_2/\partial q_1)h_2]\, h_1L_{22}/h_2 + [(\partial p_2/\partial q_2)h_1]\, h_2L_{11}/h_1\}/[(\partial p_2/\partial q_1)h_2 + (\partial p_2/\partial q_2)h_1], \quad (26)$$

for any $t \in [0, t^*]$.

The right-hand-sides of (25) - (26) are the weighted arithmetic means of h_1L_{22}/h_2 and h_2L_{11}/h_1 with strictly positive weights. Hence, this Lemma implies that discrimination moves prices in different directions unless L_{12} is large enough. Alternatively, prices move the same direction only when L_{12} is sufficiently large and price discrimination greatly increases or decreases the output of both markets. Also, by considering that $[(\partial p_2/\partial q_1)h_2] < [(-\partial p_1/\partial q_1)h_2]$ and $[(\partial p_1/\partial q_2)(-h_1)] < [(\partial p_2/\partial q_2)h_1]$ hold from (A3), if $h_1L_{22}/h_2 < h_2L_{11}/h_1$, then (25) implies (26), and if $h_1L_{22}/h_2 > h_2L_{11}/h_1$, then (26) implies (25).

Now, let us further analyze the sufficient condition for Pareto improvement of Corollary 1. From (23) - (24), the change of average price is given by

$$d[(p_1 + p_2)/2]/dt = [(\partial p_1/\partial q_1 + \partial p_2/\partial q_1)(h_2L_{12} - h_1L_{22}) + (\partial p_1/\partial q_2 + \partial p_2/\partial q_2)(h_1L_{12} - h_2L_{11})]/(2\Phi). \quad (27)$$

Under the condition of Corollary 1, $d[(p_1 + p_2)/2]/dt < 0$ because of (A3). Therefore, $dp_2/dt < 0$ holds because $dp_1/dt - dp_2/dt = 1$. From this observation and Corollary 1 and Lemma 3, we obtain the main result.

Proposition 3: *A sufficient condition for regime change from uniform pricing to discriminatory pricing to be Pareto improving with prices moving in different directions is that both* $h_1L_{22}/h_2 < h_2L_{11}/h_1$ *and* $h_1L_{22}/h_2 < L_{12} < \{[(-\partial p_1/\partial q_1)h_2]\, h_1L_{22}/h_2 + [(\partial p_1/\partial q_2)(-h_1)]\, h_2L_{11}/h_1\}/[(-\partial p_1/\partial q_1)h_2 + (\partial p_1/\partial q_2)(-h_1)]$ *hold for any* $t \in [0, t^*]$. *If both* $h_1L_{22}/h_2 < h_2L_{11}/h_1$ *and* $\{[(-\partial p_1/\partial q_1)h_2]\, h_1L_{22}/h_2 + [(\partial p_1/\partial q_2)(-h_1)]\, h_2L_{11}/h_1\}/[(-\partial p_1/\partial q_1)h_2 + (\partial p_1/\partial q_2)(-h_1)] < L_{12} < h_2L_{11}/h_1$ *hold for any* $t \in [0, t^*]$, *Pareto improvement with both prices decreasing occurs.*

We can explain Proposition 3 as follows. From (20) - (21), the change of total output is $dQ/dt = dq_1/dt + dq_2/dt = [(h_1 + h_2)L_{12} - h_2L_{11} - h_1L_{22}]/\Phi$. The change in network effects for the consumer in market 1, NE_1, is $dNE_1/dt = f_1'[(h_1 + h_2)L_{12} - h_2L_{11} - h_1L_{22}]/\Phi$. Considering that $\partial p_1/\partial q_1 = r_1' + f_1'$, $\partial p_1/\partial q_2 = f_1'$ and (23), the difference between the positive change in network effect and effect from the change in price is $dNE_1/dt - dp_1/dt = r_1'[h_1L_{22} - h_2L_{12}]/\Phi > 0$ because $r_1' < 0$ and $h_1L_{22}/h_2 < L_{12}$. By this result, the positive change in network effect dominates the effect from change in price, and the consumer in market 1 is better off even if the price increases: $dp_1/dt > 0$. If total output increases and the price of own market decreases, consumer surplus in that market obviously increases.

In the analysis so far, we only presented the conditions to obtain the results under our assumptions, but do not show that these assumptions and conditions are met under certain functional forms. In fact, if there is no network effect and $f_1 = f_1 = 0$, then $L_{12} = 0$, so the conditions of Proposition 3 are not met. Thus, in this case, Proposition 3 only represents the well-known result that Pareto improvement does not occur under independent demands with constant marginal cost. To see the effectiveness of our results, we present an example in the next section.

4. An example: linear functions

In this section, we specify a functional form that confirms the results shown in the previous section. Assume that both representative consumers have identical valuation of the intrinsic property of the good, $u_i(q_i) = aq_i - q_i^2/2$, so $r_i(q_i) = a - q_i$, but different valuations of the network, $f_i(S) = n_iS$, where n_i is non-negative and denotes the strength of network effects in market i. Without loss of generality, let $n_1 \geqq n_2$ ($\geqq 0$). Then, under a self-fulfilling expectation formation, the monopolist faces the following inverse demands:

$$p_i(q_1, q_2) = a - (1 - n_i)q_i + n_iq_j. \tag{28}$$

In this setting, the corresponding assumptions to (A1) - (A5) are given as follows: (B1) The monopolist has a capacity limit $2K = 4a/(1 - n_1 - n_2)$; (B2) $1 - n_1 > 0$, $1 - n_2 > 0$, and $(1 - n_1)(1 - n_2) - (n_1 + n_2)^2/4 = 1 - n_1 - n_2 - (n_1 - n_2)^2/4 > 0$; (B3) $1 - n_1 - n_2 > 0$, (B4) $\Pi_{ii} = -2(1 - n_i) < 0$ and $\Pi_{11}\Pi_{22} - (\Pi_{12})^2 = 4(1 - n_1)(1 - n_2) - (n_1 + n_1)^2 > 0$; (B5) $h(q_1, q_2) \equiv p_1(q_1, q_2) - p_2(q_1, q_2)$ is convex. Therefore, (B2) only matters because (B2) implies (B3) - (B5), and (B1) has virtually no effect. Also we can rewrite the latter condition of (B2) as

$$n_1 < n_2 - 2 + 2\sqrt{2(1-n_2)}. \tag{29}$$

To satisfy $n_1 \geqq n_2$, $n_2 < 1/2$ must hold. Define $n^S \equiv n_2 - 2 + 2\sqrt{2(1-n_2)}$ as the supremum of n_1, then $dn^S/dn_2 = 1 - 2/\sqrt{2(1-n_2)} < 0$.

From (28), we obtain the (direct) demands

$$q_i(p_1, p_2) = [(1 + n_i - n_j)a - (1 - n_j)p_i - n_ip_j]/(1 - n_1 - n_2). \tag{30}$$

Note that the supremum of the uniform price for opening market i is the same in both markets, $p = a$, so market closing under a regime of uniform pricing never occurs. Thus, our condition that the firm serves both markets is assured. Therefore, we can apply Proposition 3 to this setting. Considering the equalities $L_{ii} = \Pi_{ii} = -2(1 - n_i)$, $L_{12} = \Pi_{12} = n_1 + n_2$, $h_1 = -1 + n_1 - n_2$, and $h_2 = 1 + n_1 - n_2$, a simple calculation shows that $h_1L_{22}/h_2 < h_2L_{11}/h_1$ and $L_{12} < \{[(-\partial p_1/\partial q_1)h_2]\ h_1L_{22}/h_2 + [(\partial p_1/\partial q_2)(-h_1)]\ h_2L_{11}/h_1\}/[(-$

$\partial p_1/\partial q_1)h_2$ + $(\partial p_1/\partial q_2)(-h_1)]$ hold unless $n_1 = n_2$. Following from Lemma 3 and the discussion immediately after it, we know that the price of market 1 rises and the price of market 2 falls when shifting from regimes uniform pricing to third-degree price discrimination. Also, $h_1L_{22}/h_2 < L_{12}$ is equivalent to

$$n_1 > n_2 - (3 - \sqrt{17 - 16n_2})/2. \quad (31)$$

Define $n^I \equiv n_2 - (3 - \sqrt{17 - 16n_2})/2$ as the infimum of n_1 for Pareto improving, then $dn^I/dn_2 = 1 - 4/\sqrt{17 - 16n_2}$.

Now, we can calculate $n^I < n^S$ hold if and only if $0 \le n_2 < 1/2$. Then, based on the fact that $n_2 < n^I$ for $0 \le n_2 < 1/2$, we obtain the next result as a Corollary of Proposition 3

Corollary 2: *Under linear functions, the necessary and sufficient condition for regime change from uniform pricing to discriminatory pricing to be Pareto improving with prices moving in different directions is that*

$$n^I = n_2 - (3 - \sqrt{17 - 16n_2})/2 < n_1 < n_2 - 2 + 2\sqrt{2(1 - n_2)} = n^S \text{ where } 0 \le n_2 < 1/2. \quad (32)$$

Note that linearity implies that h_1L_{22}/h_2 and L_{12} from Proposition 3 are constant, so the condition (32) is not only a sufficient condition but also a necessary condition. Also, the range of n_1 become smaller as n_2 increases because $dn^S/dn_2 - dn^I/dn_2 < 0$ for $0 \le n_2 < 1/2$.

Here, we will discuss this Pareto improvement condition in more intuitive than in Proposition 3. For this purpose, let us list equilibrium outcomes derived by a simple calculation.[22]

Uniform pricing: $p^u = c + (a - c)/2$, $q_i^u = (1 + n_i - n_j)(a - c)/[2(1 - n_1 - n_2)]$,

$$Q^u = (a - c)/(1 - n_1 - n_2). \quad (33)$$

[22] We assume that the equilibrium outcomes are strictly positive. This is guaranteed if $a - c > 0$ and c is not sufficiently low.

Discriminatory pricing: $p_i^d = c + [2(1 - n_1 - n_2) + (n_i - n_j)(1 - n_i + n_j)](a - c)/D$,

$$q_i^d = (2 + n_i - n_j)](a - c)/D,\ Q^d = 4(a - c)/D, \text{ where } D = 4(1 - n_1 - n_2) - (n_1 - n_2)^2 > 0. \quad (34)$$

First, $Q^d - Q^u = (n_1 - n_2)^2(a - c)/[(1 - n_1 - n_2)D]$ holds, and (32) implies $n_1 > n_2$, then price discrimination increases the total output.[23] As shown by (19), the monopoly firm can exploit the full increment of network effects, so it has an incentive to expand the network. Then, how would this be achieved? From (30), the total demand, $Q(p_1, p_2)$, is given by

$$Q(p_1, p_2) = (2a - p_1 - p_2)/(1 - n_1 - n_2) = 2[a - (p_1 + p_2)/2]/(1 - n_1 - n_2). \quad (35)$$

Thus, it is determined by the average price. Therefore, as shown in the previous section, the monopolist lowers the average price. This fact implies that the decrement of the price in market 2 is larger than the increment of the price of market 1. Therefore, even if the own-price effect dominates the cross-price effect, $|\partial q_1/\partial p_1| > |\partial q_1/\partial p_2|$, by (A3), the output in market 1 increases if $|\partial q_1/\partial p_1| = 1 - n_2$ is not so large and $|\partial q_1/\partial p_2| = n_1$ is relatively large.

Here, let us present a numerical example where Pareto improvement occurs:

Numerical example: *Let* $a = 2$, $c = 1$, $n_1 = 0.6$ *and* $n_2 = 0.2$.

Then, $p^u = 1.5$, $p_1^d = 2$, $p_2^d = 0.75$, $3.5 = q_1^u < q_1^d = 3{,}75$, *and* $1.5 = q_1^u < q_1^d = 2.5$.

We can check that this example satisfies all assumptions, and the condition of Corollary 2 because $n^I = 0.557$ and $n^S = 0.730$. Moreover, what should be noted here is that the monopolist sets the price of market 2 below the marginal cost. This result is parallel to that of Jing (2007). He considers second-degree price discrimination with network effects, and shows that when network effects are sufficiently strong, the monopolist sells the low-quality product below marginal cost to expand its network and gain more revenue from the high-quality product by setting its price high. Similarly, in our model, the monopolist could offer the good in market 2 at price below marginal cost to exploit the effects of network expansion.

[23] This total output effect is consistent with Layson's (1998) condition of linear demands case —$\partial q_2/\partial p_1$ greater than $\partial q_1/\partial p_2$ implies increasing total output.

From (34), solving $p_2^d < c$, we can show that the condition for below marginal cost pricing coincides with (32). Summarizing the analysis so far, we obtain the following result.

Proposition 4: *Under linear functions, if and only if $n^I < n_1 < n^S$, Pareto improving occurs with the price of market 1 being larger than under uniform price and the price of market 2 being smaller than marginal cost.*

In the end of this section, let us consider aggregate consumer surplus. From (17), the difference in aggregate consumer surplus between the discriminatory pricing regime and the uniform pricing regime is given by $CS^d - CS^u = [(q_1^d)^2 + (q_2^d)^2 - (q_1^u)^2 - (q_2^u)^2]/2$. Unfortunately, because of the complexity of calculation, we cannot obtain an explicit condition for improving aggregate consumer surplus. However, by using the bounds of Proposition 2, we have a sufficient condition. A simple calculation shows that the upper bound is always positive and lower bound, $\sum_i\{[r_i(q_i^u) - r_i(q_i^d)]q_i^u\}$, is positive if and only if $(1 - n_2)/3 < n_1$. Also, we can show $(1 - n_2)/3 < n^I$ for $0 \leq n_2 \leq 1/2$. Therefore, the bounds provide us with useful information that when $(1 - n_2)/3 < n_1 < n^I$ hold, aggregate consumer surplus will increase even if Pareto improvement does not occur.

5. Concluding remarks

In general, a monopolist selling in two separate markets that switches its pricing regime from uniform pricing to third-degree price discrimination raises its price in one market and lowers its price in the other market. Therefore, it has been considered that third degree price discrimination decreases consumer surplus of the market in which the price rises and so implies that the change in pricing regime is not Pareto improving. However, this paper shows that third-degree price discrimination could be beneficial for all consumers and the monopolist when network effects exist, even if its price rises in one market.

To obtain this result, in Lemma 1 we first clarify the difference in the measurement of consumer surplus depending on whether there are network effects. Then, based on Lemma 1, we present the decomposition of social welfare, and in Proposition 1 obtain the necessary condition for social welfare to be increased by price discrimination, and in Proposition 2 the necessary condition and sufficient condition for aggregate consumer surplus to be improved. Subsequently, in Proposition 3 we get a sufficient condition for Pareto improvement under the general setting. Moreover, by specifying functions as linear, we confirm how the results of Propositions 2 and 3 work effectively. Furthermore, Proposition 4 shows that Pareto improvement with below-marginal-cost pricing occurs under the simple linear model.

Our limitations are as follows. First, although Proposition 3 could be applicable to other general functions, we present only a linear example. Second, we do not pursue the condition for all consumers to be worse-off although Lemma 2-(ii) touches on this possibility. In contrast to our Pareto improving mechanism, if the total output decreases by price discrimination, the consumer in a market whose price falls can be worse-off. It seems that this happens under a linear model if consumers' valuations of the intrinsic property differ and the price in the market with weaker network effects rises. Third, we need to connect our findings to empirical research. In particular, selling below marginal cost is considered dumping and subject to criticism, but our finding implies that below-marginal-cost pricing improves the welfare of all economic agents. Therefore, to choose regulation policies, it will be necessary to clarify what kind of situation has given rise to below-marginal-cost pricing.

Finally, let us consider the empirical relevance of our findings. We show that Pareto improvement could occur if the strength of network effects differs. Therefore, in markets for software, professionals value network effects higher than do general users because professionals will think that they can exert their ability in a wider range of situations if the software spreads broadly. In this situation, third-degree price discrimination could bring about a Pareto improvement.

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