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The More Firms, the More Insufficient Entry

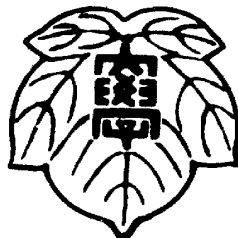
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Discussion Paper No. 16-2, May 2016

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# The More Firms, the More Insufficient Entry

Takeshi Ikeda\*

## Abstract

Mukherjee (2012) shows that entry is socially insufficient when entrants are rather minor in a Stackelberg model. The insufficiency results from the business-creation effect; that is, the entrants of minor firms enhance the output of a major leader. In this paper, we consider more than one Stackelberg leader and find that the existence of multiple leaders encourages insufficient entry because the business-creation effect increases. Moreover, we show that paradoxically, the more firms present, the more insufficiency of entry we have under a constant marginal cost when the number of leaders is past a certain level.

**Keywords:** Excess entry; Insufficient entry; Stackelberg model; Multiple leaders

**JEL:** L11; L13

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## 1. Introduction

Many papers have considered excess and insufficient entry of firms in a Cournot competition. The seminal researches that treat excess entry are Mankiw and Whinston (1986) and Suzumura and Kiyono (1987). They find that excess entry may occur when firms have scale economies.<sup>1</sup> On the other hand, Matsumura (2000) shows that insufficient entry may take place if the marginal cost is increasing. Moreover, Matsumura and Okamura (2006) also shows that insufficient entry may occur when entry cost is large by using a circular-city model.<sup>2</sup>

Recently, Ino and Matsumura (2012) finds that, by using a Stackelberg model, excess entry takes place. In contrast, Mukherjee (2012) shows that insufficient entry occurs when an entrant follower is rather minor. The result is opposite to one that obtained in Ino and Matsumura (2012). Note that in the model of Mukherjee (2012), there is only one major Stackelberg leader. The insufficiency results from the business-creation effect; that is, entry of a minor Stackelberg follower enhances the output of a major Stackelberg leader. The logic is very simple and new; therefore, we can apply it to a lot of studies.

In reality, many markets consist of major leaders and minor followers. For example, the beer industry in Japan consists of four major firms, Kirin, Asahi, Suntory and Sapporo, and many other minor firms.

Then, assuming the presence of multiple leaders, how will entry inefficiency change? Mukherjee (2012) notes that the presence of more than one leader also has a business-creation effect; therefore, a similar result might be assumed from using more leaders. However, multiple leader firms will steal each other's business, thereby weakening the business-creation effect. In addition, the number of leaders impacts the number of follower entrants.

In this paper, we consider  $m$  Stackelberg leaders and free entry Stackelberg followers and find that the presence of more leaders increases insufficiency of entry. In other words, under a constant marginal cost, increasing the number of leader firms enhances insufficiency of entry. Moreover, we show that paradoxically, the more firms present, the more insufficiency of entry we have under a constant marginal cost when the number of leaders is past a certain level.

## 2. The model

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<sup>1</sup> See also Suzumura (1995, 2012).

<sup>2</sup> See also Gosh and Morita (2007a, 2007b) and Mukherjee (2010).

We consider  $m$  Stackelberg leaders and free entry Stackelberg followers. The timing of the game is the same as that of Mukherjee (2012): at stage 1,  $m$  leaders enter the market, at stage 2, followers decide whether to enter the market, at stage 3,  $m$  leaders determine their outputs, and at stage 4, the follower entrants determine their outputs. The inverse demand function is  $P = a - Q$ , where the notations have the usual meanings. Following Mukherjee (2012), we assume the marginal cost of leaders is constant, zero; on the other hand, the followers' marginal cost is positive constant,  $c$ . To obtain the positive output of followers, we suppose  $c < a/(mn + m + 1)$  ( $\equiv c_{\max}$ ). All the firms need fixed cost  $k$  to enter the market.

When  $n$  followers enter the market, follower  $j$ 's profit function is

$$\pi_j = (a - Q - c)x_j - k, (j = 1, 2, \dots, n) \quad (1)$$

where  $x_j$  is the output of follower  $j$ . From the first-order condition, we have

$$x_j = \frac{a - c - X}{n + 1} \quad (2)$$

where  $X$  is the total output of leaders.

On the other hand, leader  $i$ 's profit function is

$$\Pi_i = (a - Q)X_i - k, (i = 1, 2, \dots, m) \quad (3)$$

where  $X_i$  is the output of leader  $i$ . From the first-order condition and (2), we get

$$X_i^* = \frac{a + nc}{1 + m} \quad (4)$$

where  $X_i^*$  is the equilibrium output of leader  $i$ . From (2) and (4), we obtain

$$x_j^* = \frac{a - c(mn + m + 1)}{(m + 1)(n + 1)} \quad (5)$$

where  $x_j^*$  is the equilibrium output of follower  $j$ . The equilibrium total output of leaders and followers is

$$Q^* = \frac{am(n + 1) + n(a - c)}{mn + m + n + 1}. \quad (6)$$

Then, the equilibrium profits of follower  $j$  and leader  $i$  and consumer surplus are, respectively:

$$\pi_j^* = \frac{(a - c(mn + m + 1))^2}{(m + 1)^2(n + 1)^2} - k \quad (7)$$

$$\Pi_i^* = \frac{(a + cn)^2}{(m + 1)^2(n + 1)} - k \quad (8)$$

$$CS^* = \frac{(cn - a(mn + m + n))^2}{2(1 + m)^2(1 + n)^2}. \quad (9)$$

Free entry of followers leads to  $\pi_j^* = 0$ , i.e.,

$$\frac{(a - c(mn + m + 1))^2}{(m + 1)^2(n + 1)^2} = k. \quad (10)$$

From (10), we have the equilibrium number of followers,  $n^*$ . Ignoring the integer problem, we can totally differentiate (10). Therefore, we obtain

$$\frac{dn^*}{dm} = -\frac{c(n + 1)(a - c(mn + m + 1)) + k(m + 1)(n + 1)^2}{cm(a - c(mn + m + 1)) + k(m + 1)^2(n + 1)} < 0. \quad (11)$$

(11) shows that an increase in leader firms reduces follower entrants.

We now consider the socially optimum number of followers. Social welfare is the sum of the firms' profits and consumer surplus, that is,  $W = m\Pi_i + n\pi_j + CS$ . From (7), (8) and (9), we obtain

$$W^* = \frac{2m(n + 1)(a + cn)^2 + 2n(a - c(mn + m + 1))^2 + (cn - a(mn + m + n))^2}{2(m + 1)^2(n + 1)^2} - (m + n)k \quad (12)$$

where  $W^*$  is the equilibrium social welfare. Differentiating  $W^*$  with respect to  $n$ , we have

$$\begin{aligned} \frac{\partial W^*}{\partial n} &= 0 \\ \Leftrightarrow \frac{a^2 - ac(mn + m + 2) + c^2(1 + m^2(1 + n)^3 + m(n^3 + 3n^2 + 4n + 2))}{(m + 1)^2(n + 1)^3} - k &= 0. \quad (13) \end{aligned}$$

From (13), we know the socially optimum number of followers.<sup>3</sup>

From (10) and (13), we have

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<sup>3</sup> The second-order condition  $\partial^2 W^* / \partial n^2 < 0$  is satisfied when  $0 < c < a / (mn + m + 1)$ .

$$\frac{a^2 - ac(mn^* + m + 2) + c^2(1 + m^2(1 + n^*)^3 + m(n^{*3} + 3n^{*2} + 4n^* + 2))}{(m+1)^2(n^* + 1)^3} - \frac{(a - c(mn^* + m + 1))^2}{(m+1)^2(n^* + 1)^2} \equiv z \quad (14)$$

$z$  is increasing in  $c$  when  $0 < c < a/(mn + m + 1)$ , and if it is positive (negative), the number of followers is below (above) the socially optimum number.

Let us consider  $c^*$  that satisfies  $z = 0$ . In other words, entry is socially insufficient (excessive) when  $c^* < c < c_{\max}$  ( $0 < c < c^*$ ). Mukherjee (2012) has already found that  $c^*$  exists when  $m = 1$ . Therefore, we consider how the value of  $c^*$  changes when  $m$  moves. Note that  $n^*$  depends on  $c$  and  $m$ . From (14), we have

$$\frac{dz}{dm} = \frac{ac^2 - (c + \sqrt{k})(c^2 - c\sqrt{k}(1+m)^2 - k(1+m)^2)}{(a-c)(1+m)^2} \begin{matrix} > 0 \\ < 0 \end{matrix}$$

$$\Leftrightarrow (a-c)c^2 + (c + \sqrt{k})(c\sqrt{k}(1+m)^2 + k(1+m)^2) - c^2\sqrt{k} \begin{matrix} > 0 \\ < 0 \end{matrix}$$

Since  $a > c$ , we obtain  $dz/dm > 0$ , that is, an increase in  $m$  shifts  $z(c)$  upward. Noting that  $z(c)$  is increasing in  $c$  when  $0 < c < a/(mn + m + 1)$ ,  $dz/dm > 0$  shows that an increase in  $m$  lowers  $c^*$ . Therefore, we have the following proposition.

**Proposition 1:** *An increase in the number of leader firms lowers  $c^*$ , i.e., the insufficient entry takes place more likely when the number of leader is larger.*

Assuming that  $a = 100$ ,  $c = 2$ , and  $k = 2$ , we have figure 1 and table 1, which demonstrate proposition 1. Proposition 1 indicates that under a constant marginal cost, an increase in leader firms enhances insufficiency of entry. The logic is as follows: From (4) we know an entrant of a minor follower firm enhances the output of leader  $i$ . It is called the “business-creating effect” in Mukherjee (2012). Note that business creating occurs only when the marginal cost of a leader is lower than that of a follower. If the marginal cost of followers is rather large, the effect is also large and exceeds the business-stealing effect among followers. Therefore, new follower entrants enhance consumer surplus while reducing producer surplus. In our model, we know from (4) that an increase in the number of leaders reduces *individual* business-creating effects but enhances *total* business-creating effects. Moreover, from (11), we know that the presence of more leaders reduces the free entry equilibrium number of followers. Therefore, insufficient entry is realized even when the cost difference between a leader

and follower is small.

In addition, from (11) and proposition 1, we obtain the following proposition.

**Proposition 2:** *The more leaders present, the more firms operate in the whole market, nevertheless entry is more insufficient when  $dn^*/dm > -1$ .*

From (10), we know the equilibrium number of followers, and assuming  $a = 100$ ,  $c = 2$ , and  $k = 2$ , we obtain table 2, which demonstrates proposition 2. From table 2, we see that an increase in leaders causes the number of firms in the whole market to increase when there are five or more leaders; that is, paradoxically, the more firms present, the more insufficiency of entry we have under a constant marginal cost.

### 3. Conclusion

Typically, the excess entry theorem comes into existence as a result of enhancing producer surplus and reducing consumer surplus; that is, when firms exit the market, although total output reduces, incumbent firms increase production and producer surplus increases because of scale economies. On the other hand, insufficient entry in both Mukherjee (2012) and this paper is shown to be a result of enhancing consumer surplus and reducing producer surplus. Moreover, we find that insufficient entry is likely to occur when multiple leaders exist. Therefore, policy makers need to be careful that they adopt a policy of market entry regulation.

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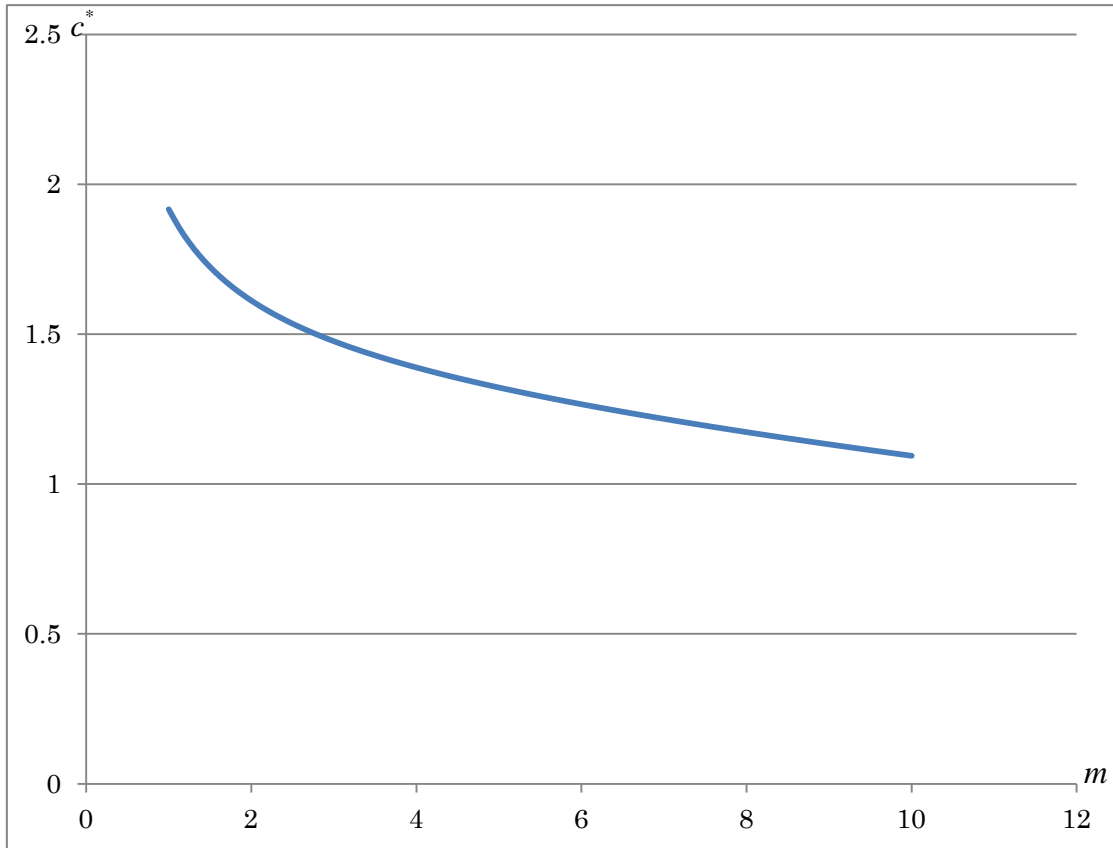
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## References

- Gosh, A and H. Morita, 2007a, "Social desirability of free entry: A bilateral oligopoly analysis," *International Journal of Industrial Organization*, 25, 925-934.
- Gosh, A and H. Morita, 2007b, "Free Entry and Social Efficiency under Vertical Oligopoly," *RAND Journal of Economics*, 38, 541-554.
- Ino, H. and T. Matsumura, 2012, "How many firms should be leaders? Beneficial concentration revisited," *International Economic Review*, 53, 1323-1340.
- Mankiw, A. and M. Whinston, 1986, "Free entry and social inefficiency," *RAND Journal of Economics*, 17, 48-58.
- Matsumura, T., 2000, "Entry regulation and social welfare with an integer problem," *Journal of Economics*, 71, 47-58.
- Matsumura T. and M. Okamura, 2006, "Equilibrium number of firms and economic welfare in a spatial price discrimination model," *Economics Letters*, 90, 396-401.
- Mukherjee, A., 2010, "Endogenous cost asymmetry and insufficient entry in the absent of scale economies," *Journal of Economics*, 106, 75-82.
- , 2012, "Social efficiency of entry with market leaders," *Journal of Economics and Management Strategy*, 21, 431-444.
- Suzumura, K., 1995, *Competition, Commitment and Welfare*, New York, USA: Oxford University Press.
- , 2012, "Excess entry theorems after 25 years," *Japanese Economic Review*, 63, 152-170.
- and K. Kiyono, 1987, "Entry barriers and economic welfare," *Review of Economic Studies*, 54, 157-167.

**Figure 1. Relationship between  $m$  and  $c^*$ .**



**Table 1. The approximate value of  $c^*$ .**

	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 10$
$c^*$	1.92	1.61	1.48	1.39	1.32	1.27	1.09

**Table 2. The approximate value of  $n^*$ .**

	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$
$n^*$	19.30	10.89	7.41	5.50	4.30	3.47	2.87